B.Sc - Part I

Question based on properties of definite integrals.

1.) Show that $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$.

Solⁿ Here $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

OR $I = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx$

OR $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ — (2)

Adding (1) & (2) we get

$2I = \int_0^{\pi/2} \left[\frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right] dx$

$= \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \int_0^{\pi/2} dx = [x]_0^{\pi/2}$

$= \pi/2$

$\therefore I = \pi/4$

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Evaluate $\int_0^{\pi/2} \log \sin x \, dx$

Soln Here $I = \int_0^{\pi/2} \log \sin x \, dx$ — (i)

or, $I = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx$ — (ii)

$= \int_0^{\pi/2} \log \cos x \, dx$

Adding (1) & (2), we get

$2I = \int_0^{\pi/2} (\log \sin x + \log \cos x) dx$

$= \int_0^{\pi/2} \log(\sin x \cos x) \, dx$

$= \int_0^{\pi/2} \log\left(\frac{2 \sin x \cos x}{2}\right) dx.$

$= \int_0^{\pi/2} \log \sin 2x \, dx - \int_0^{\pi/2} \log 2 \, dx$

$= I_1 - \pi/2 \log 2$

where $I_1 = \int_0^{\pi/2} \log \sin 2x \, dx$

Put $2x = t \therefore 2dx = dt$
when $x = 0$, $t = 0$ & when $x = \pi/2$,
$t = \pi$

$$\therefore I_1 = \tfrac{1}{2} \int_0^{\pi} \log \sin t \, dt$$

$$= \frac{1}{2} \int_0^{2 \cdot \pi/2} \log \sin t \, dt$$

$$= \frac{1}{2} \cdot 2 \int_0^{\pi/2} \log \sin t \, dt = I$$

$$\therefore 2I = I_1 - \pi/2 \log 2$$

or $2I = I - \dfrac{\pi}{2} \log 2$

$$I = -\frac{1}{2} \pi \log 2 \text{ or } \frac{\pi}{2} \log \tfrac{1}{2}$$